Quantum, Classical and Symmetry Aspects of Max Born Reciprocity

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Topics

1. Reciprocal Symmetry and Maximum Tension Principle
2. Maximum Force and Newton gravity
3. Extended Phase Space (QTPH) as a Basic Manifold
4. Complex Lorentz group with Real Metric as Group of Reciprocal Symmetry
5. One-Particle Quasi-Newtonian Reciprocal-Invariant Hamiltonian dynamics
6. Canonic Quantization: Dirac Oscillator as Model of Fermion with a Plank mass

Born Reciprocity version

1. Reciprocity Transformations (RT):

$$\frac{x_\mu}{q_e} \to \frac{p_\mu}{p_e}, \quad \frac{p_\mu}{p_e} \to -\frac{x_\mu}{q_e}.$$

2. Lorentz and Reciprocal Invariant quadratic form:

$$S_B^2 = \frac{1}{q_e^2} x^\mu x_\mu + \frac{1}{p_e^2} p^\mu p_\mu, \quad \eta_{\mu\nu} = \mathrm{diag}\{1, -1, -1, -1\}$$

3. x_μ, p_μ are quantum-mechanical canonic operators:

$$[x_\mu, p_\nu] = i\hbar\eta_{\mu\nu}$$

4. Phenomenological parameter according definition $p_e = Mc$ so that

$$q_e = \hbar/Mc = \Lambda_c \quad \text{(Compton length)}.$$

   The mass M is a free model parameter.

5. Two dimensional constants q_e[length], p_e[momentum] connected by corellation:

$$q_e p_e = \hbar \quad \text{(Plank constant)}.$$

Relativistic Oscillator Equation (ROE)

Selfreciprocal-Invariant Quantum mechanical Equation:

$$\left(-\frac{\partial^2}{\partial\xi^\mu\partial\xi_\mu}+\xi^\mu\xi_\mu\right)\Psi(\xi)=\lambda_B\Psi(\xi),$$

where $\xi^\mu = x^\mu/\Lambda_c$.
The (ROE) symmetry $\approx U(3,1)$. There are solution corresponding linear discrete mass spectrum.

Maximum Tension Principle (MTP)

The space-time and momentum variables are to be just the same dimentions of a quantity. It is necessary to have the universal constant with dimention: momentum/length, or equivalently: mass/time, energy/time, momentum/time. In reality: MTP [Gibbons, 2002]
Gibbons Limit:

$$\text{Maximum force} \quad \boldsymbol{F_{max} = \left(\frac{dp}{dt}\right)_{max} = \frac{c^4}{4G_N} = F_G},$$

$$\text{Maximum power} \quad \boldsymbol{P_{max} = \left(\frac{dE}{dt}\right)_{max} = \frac{c^5}{4G_N} = P_G}.$$

[momentum]/[length] – Universal constant

$$\boldsymbol{æ_0 = \frac{p_e}{q_e} = \frac{c^3}{4G_N}} \quad \text{[LMT, 1974, 2003]}$$

It is evident: $\boldsymbol{æ_0 = c^{-1}F_{max} = c^{-2}P_{max}}$. All these values are essentially classic (does not contain Plank constant $\hbar$).

Born Reciprocity and Gibbons Limit

Let the parameter $\boldsymbol{a}$ to be some fixed (nonzero!) value of action $\boldsymbol{S}$. We replace initial Born's relation

$$\boldsymbol{q_e p_e} = \hbar \quad \text{by} \quad \boldsymbol{\pi q_e p_e = a}$$

without beforehand action discretness supposition. We demand the nonzero area $\boldsymbol{S_{fix} = \pi q_e p_e}$ of the phase plane only.
The second suggestion is:

$$\boldsymbol{æ_0 = \frac{p_e}{q_e} = \frac{c^3}{4G_N}} \quad \text{(universal constant).}$$

Now the parameters q_e, p_e are defined separately

$$q_e = \sqrt{\frac{a}{\pi æ}}, \quad p_e = \sqrt{\frac{aæ}{\pi}}$$

Evidently: the phenomenological constants $E_e = p_e c$ [Energy] and $t_e = c^{-1} q_e$ [time] satisfy the conditions

$$E_e t_e = \pi a, \quad \frac{E_e}{t_e} = c^2 æ = \frac{c^5}{4G_N}$$

Under Born's parametrization

$$p_e = Mc, \quad q_e = \frac{2MG_N}{c^2} = r_g(M)$$ – gravitational radius.

General mass-action correlation

$$M^2 = \frac{ac}{4\pi G_N}.$$

It is valid as in classical as in quantum case. Under supposition when $a_{min} = h$ we receive

$$M^2 = \frac{hc}{4\pi G_N} = \frac{1}{2} M_{Pl}^2, \quad M_{Pl} = \sqrt{\frac{\hbar c}{G_N}}$$ – Plank mass.

Maximum Force and Modified Newton Gravity

$$\text{(NG):} \quad \left|\underline{f}\right| = G_N \frac{mM}{r^2} \equiv \frac{c^4}{4G_N} \frac{2mG_N}{c^2} \frac{2MG_N}{c^2} \frac{1}{r^2}$$

$$= F_G \frac{r_g R_g}{r^2} \equiv F_G \frac{r_0^2}{r^2} \quad (r \geqslant 0, \quad r_0 = \sqrt{r_g R_g})$$

$$\text{(ModNG):} \quad \left| f_N^{mod} \right| = F_G \frac{r_0^2}{r^2} \quad (r \geqslant r_0), \quad \left| f_N^{mod} \right|_{r=r_0}^{max} = F_G$$

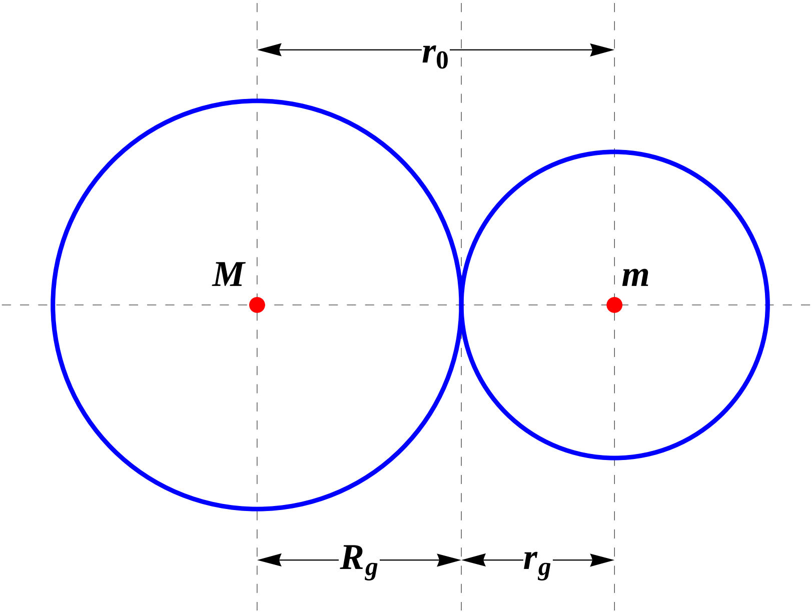
The corresponding gravitational potential Energy

$$U_{gr}^{mod} = -F_G \frac{r_0^2}{r}$$

possess a minimum under $r = r_0$

$$U_{gr}^{mod}(min) = -F_G r_0 = -\frac{c^4}{4G_N} \frac{2G_N}{c^2} \sqrt{mM} = -\frac{c^2}{2} \sqrt{mM}$$

We have dealing (instead of point-like masses) with a pair spacely extended simple massive gravitating objects like the elastic balls or drops with a high surface tension.



$$E_{rest}(M,m) = Mc^2 + mc^2 - \frac{1}{2}\sqrt{mM}c^2$$

The whole energy such a "binary" system is as follows:

$$\boldsymbol{E(M,m) = Mc^2 + mc^2 - \frac{1}{2}\sqrt{mM}c^2}$$

$$\boldsymbol{= (M+m)c^2\left\{1 - \frac{1}{2}\sqrt{\frac{\mu}{M+m}}\right\}},$$

where $\boldsymbol{\mu = mM/(m+M)}$.

$$\boldsymbol{Mc^2, \quad mc^2, \quad (M+m)c^2} \text{ – rest energy}$$

$$\boldsymbol{\Delta E = \frac{1}{2}\sqrt{\mu(M+m)}c^2 = \frac{1}{2}\sqrt{mM}c^2} \text{ – energy corresponding}$$

the "mass defect" $\boldsymbol{\Delta m = \frac{1}{2}\sqrt{mM}}$

“Gravitational fusion” picture

$$\lambda = \frac{\text{[eliminated energy]}}{\text{[rest energy]}} = \frac{\Delta E}{(M+m)c^2}$$
$$= \frac{\sqrt{mM}}{2(M+m)} = \frac{1}{2}\frac{\sqrt{\delta}}{1+\delta},$$

where

$$\delta = \frac{m}{M}, \quad \delta_0 \leqslant \delta \leqslant 1, \quad \delta_0 = \frac{M_{min}}{M_{max}}$$

$$\text{Maximum} \quad \lambda_{max} = \frac{\Delta E_{eliminated}}{E_{rest}} = \frac{1}{4}$$

$$\text{under} \quad \delta = 1 \quad (m = M)$$

Such a “fusion” of two equal rest mass M lead to mass $= 3M/2$, the Energy $\Delta E = Mc^2/2$ eliminate $(3:1)$.

Extended Phase Space as a Basic Manifold

Space of states (QTP) and Energy (H) (sec [J. Synge, 1962]) in general: $\mathbf{2+2N}$ dimensions. The metric is

$$\eta_{AB} = \mathrm{diag}\{\overbrace{\mathbf{1, -1, -1, \ldots, -1}}^{N}\}$$

(X, Y) – pair of conjugated pseudoeuclidean vectors X_A, Y_A, $A, B = \mathbf{0, 1, \ldots}, N$ ($N + \mathbf{1}$ dimention).
The Poisson brackets for $\varphi(X, Y)$, $f(X, Y)$,

$$\{\varphi, f\}_{A,B} = \sum_{k=0}^{N} \left\{ \frac{\partial \varphi}{\partial X^k} \frac{\partial f}{\partial Y_k} - \frac{\partial \varphi}{\partial Y^k} \frac{\partial f}{\partial X_k} \right\}$$

so that

$$\{X_A, Y_A\}_{A,B} = \eta_{AB}$$

Dynamic System in (QT, PH) is defined, when the Energy Surface ($2N+1$ dimention)

$$\Omega(X, Y) = 0$$

is defined. Solving this equation on Y_0 we obtain Energy equation

$$Y_0 = F(X_k, Y_l, X_0) \quad k, l = 1, 2, \ldots, N$$

The corresponding Hamilton function of the System is by def:

$$\mathcal{H} \equiv Y_0 = \mathcal{H}(X_k, Y_l, Y_0)$$

We shall consider $(2 + 2 \cdot 3)$ – dimensional version (QT, PH)

$$A, B \to \mu, \nu = 0, 1, 2, 3, \quad \eta_{\mu\nu} = \mathrm{diag}\{1, -1, -1, -1\},$$

X_μ, X_μ are real 4-dimensional pseudoeuclidean $SO(3,1)$ – vectors.

Complexification of (QT, PH)

Let us introduce the complex 4-vector $\boldsymbol{Z^\mu}$ according definition:

$$\boldsymbol{Z^\mu = X^\mu + iY^\mu}, \quad \boldsymbol{\eta_{\mu\nu}} = \mathrm{diag}\boldsymbol{\{1, -1, -1, -1\}}$$

and define the real norm

$$\boldsymbol{Z^\mu Z^*_\mu = Z^\mu \eta_{\mu\nu} Z^{\mu *}}$$

$$\boldsymbol{= |Z_0|^2 - |Z_1|^2 - |Z_2|^2 - |Z_3|^2} = \mathrm{inv}$$

Complex Lorentz Group with a real metric (Barut Group [Barut 1964])

The group of $\mathbf{4 \times 4}$ complex matrices $\boldsymbol{\Lambda}$ of transformations

$$\boldsymbol{Z' = \Lambda Z, \quad Z = X + iY}$$

satisfying the condition

$$\boldsymbol{\Lambda \eta \Lambda^\dagger = \eta}, \quad (\dagger - \text{hermitien conjugation})$$

Notice: the diagonal matrices $\boldsymbol{\Lambda_R}$ and $\bar{\boldsymbol{\Lambda}}_{\boldsymbol{R}} \in (\boldsymbol{BG})$

$$\boldsymbol{\Lambda_R = -iI_4 = -i}\mathrm{diag}\{\mathbf{1, 1, 1, 1}\}$$

and

$$\bar{\boldsymbol{\Lambda}}_{\boldsymbol{R}} = \boldsymbol{-i\eta_{\mu\nu} = -i}\mathrm{diag}\{\mathbf{1, -1, -1, -1}\}$$

Metric invariant (no positive-defined)

$$\boldsymbol{Z^\mu Z^*_\mu = |Z_0|^2 - |Z_1|^2 - |Z_2|^2 - |Z_3|^2} = \mathrm{inv}$$

Three possibilities: $\boldsymbol{|Z|^2 \gtrless 0}$, $\boldsymbol{|Z|^2 = 0}$ – isotropic case

$SU(3,1)$ – *subgroup structure*

(QTPH)

$$QT = \{\underline{r}, ct\} = x^\mu$$

$$PH = \{\underline{p}, E/c\} = p^\mu$$

$$\vec{\beta}_v = \vec{v}/c$$

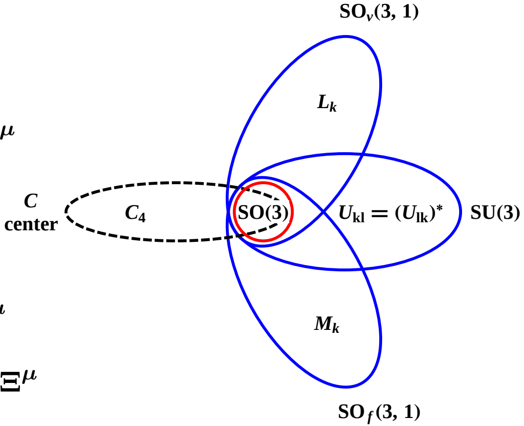
(PTQH)

$$PT = \{\underline{p}, F_0 t\} = \Pi^\mu$$

$$QH = \{\underline{r}, E/F_0\} = \Xi^\mu$$

$$\vec{\beta}_f = \vec{f}/F_0$$

$$F_0 = F_{max}$$



$$k, l = 1, 2, 3$$

Reciprocal Invariant S_B^2 as BG metric invariant $Z^\mu Z_\mu^*$

By def: $Z_\mu = X_\mu + iY_\mu$

$$X_\mu = \frac{1}{q_e} x_\mu = \frac{1}{q_e}\{x_0, x_k\}, \quad Y_\mu = \frac{1}{p_e} p_\mu = \frac{1}{p_e}\{p_0, p_k\}.$$

Then:

$$S_B^2 = \frac{1}{q_e^2}\left(x_0^2 - \sum_{k=1}^{3} x_k^2\right) + \frac{1}{p_e^2}\left(p_0^2 - \sum_{k=1}^{3} p_k^2\right) =$$

$$= \frac{x_0^2}{q_e^2} + \frac{p_0^2}{p_e^2} - \sum_{k=1}^{3}\left(\frac{x_k^2}{q_e^2} + \frac{p_k^2}{p_e^2}\right) = |Z_0|^2 - \sum_{k=1}^{3} |Z_k|^2 = Z^\mu Z_\mu^*$$

Let x_μ, p_μ to be 4-vectors under $SO_v(3,1)$-transformations

Notice! There exist alternative:

$$\bar{Z}_\mu = \bar{X}_\mu + i\bar{Y}_\mu$$

where

$$\bar{X}_\mu = \Xi_\mu \overset{\text{def}}{=} \left\{\frac{x_0}{q_e}, \frac{p_k}{p_e}\right\}, \quad \bar{Y}_\mu = \Pi_\mu \overset{\text{def}}{=} \left\{\frac{p_0}{p_e}, \frac{x_k}{q_e}\right\}.$$

It is evident:

$$|\bar{Z}_\mu|^2 = \Xi^\mu \Xi_\mu + \Pi^\mu \Pi_\mu = \frac{x_0^2}{q_e^2} - \sum_{k=1}^{3} \frac{p_k^2}{p_e^2} + \frac{p_0^2}{p_e^2} - \sum_{k=1}^{3} \frac{x_k^2}{q_e^2} =$$

$$= \frac{1}{q_e^2} x^\mu x_\mu + \frac{1}{p_e^2} p^\mu p_\mu = |Z_\mu|^2.$$

The natural supposition: Ξ_μ and Π_μ are 4-vectors under $SO_f(3,1)$-transformations

Selfreciprocal Case: $S_B^2 = 0$

Quadratic form $S_B^2 = Z^\mu Z_\mu^*$ is not positive defined
Three possibilities $Z^\mu Z_\mu^* = 0$, $Z^\mu Z_\mu^* \gtrless 0$.
Especial interesting is:

$$S_B^2 = \frac{1}{p_e^2} p^\mu p_\mu + \frac{1}{q_e^2} x^\mu x_\mu = 0$$

Two possibilities

1. $$p^\mu p_\mu = x^\mu x_\mu = 0$$

   (light cone, massless particles, standart clock synchronisation)

2. $$x^\mu x_\mu = -q_e^2, \quad p^\mu p_\mu = p_e^2,$$

   $$q_e = r_g(m_e) = \frac{2 m_e G_N}{c^2}, \quad p_e = m_e c.$$

BG: some kinematic outcome

(1) Reciprocal-Invariant time T_R

$$z_\mu = x_\mu + i\text{æ}_0^{-1} p_\mu, \quad \text{æ}_0 = \frac{p_e}{q_e} = \frac{c^3}{4G_N}$$

$$dT_R \overset{\text{def}}{=} c^{-1}\sqrt{z^\mu z^*_\mu}$$

$$= c^{-1}\sqrt{dx_0^2 - d\underline{r}^2 + \text{æ}_0^{-2}(dp_0^2 - d\underline{p}^2)}$$

$$= dt\sqrt{1 - \underline{\beta}_v^2 - \underline{\beta}_f^2 + \underline{\beta}_v^2 \underline{\beta}_f^2 \cos^2\alpha}$$

$$= dt\sqrt{\left(1 - \underline{\beta}_v^2\right)\left(1 - \underline{\beta}_f^2\right) - \left(\underline{\beta}_f \times \underline{\beta}_v\right)^2}.$$

where

$$\underline{\beta}_v = \frac{\dot{\underline{r}}}{c}, \quad \underline{\beta}_f = \frac{\dot{\underline{p}}}{F_G}, \quad F_G = \frac{c^4}{4G_N}, \quad (\dot{}) = \frac{d}{dt},$$

t – is the laboratory time (comoving observer time).
α – is the angle between $\dot{\underline{r}}$ and $\dot{\underline{p}}$.

(2) In the comoving $\boldsymbol{RF}$:

$$dT_R(\beta_v = 0) = dt\sqrt{1 - \underline{\beta}_v^2}$$

There must exist an extra time retardation.

(3) $\cos\alpha = \pm 1, \quad (\underline{\beta}_f \times \underline{\beta}_v) = 0$

$$dT_R^{\parallel} = dt\sqrt{1 - \underline{\beta}_v^2}\sqrt{1 - \underline{\beta}_f^2}$$

(4) $\cos\alpha = 0, \quad (\underline{\beta}_f \times \underline{\beta}_v)^2 = \underline{\beta}_f^2 \cdot \underline{\beta}_v^2$

$$dT_R^{\perp} = dt\sqrt{1 - \underline{\beta}_v^2 - \underline{\beta}_f^2} \neq dT_R^{\parallel}$$

(5) $\cos\alpha = \pm\sqrt{2}/2$,

$$dT_R^{\parallel} = dT_R^{\perp}$$

(6) The upper limit of $|\dot{\underline{r}}|$ and $|\dot{\underline{p}}|$ in the expression for dT_R:

$$|\dot{\underline{r}}|_{max} = c/\sqrt{2}, \quad |\dot{\underline{p}}|_{max} = F_G/\sqrt{2}$$

Complex Lorentz Boost Transformations

Let us represent a $\boldsymbol{BG}$-transformations using the following parametrization:

$$\boldsymbol{\Lambda}(\boldsymbol{\beta}) = \begin{pmatrix} \boldsymbol{\Gamma} & \boldsymbol{\Gamma} \cdot \underline{\boldsymbol{\beta}} \\ \boldsymbol{\Gamma} \cdot \underline{\boldsymbol{\beta}}^* & \boldsymbol{I_3} + \frac{\underline{\boldsymbol{\beta}} \cdot \underline{\boldsymbol{\beta}}^*}{|\underline{\boldsymbol{\beta}}|^2} (\boldsymbol{\Gamma} - \mathbf{1}) \end{pmatrix}, \quad \boldsymbol{\Lambda}(\boldsymbol{\beta}) \boldsymbol{\eta} \boldsymbol{\Lambda}^\dagger(\boldsymbol{\beta}) = \boldsymbol{\eta}$$

where $\underline{\boldsymbol{\beta}} = \underline{\boldsymbol{\beta}}_{\boldsymbol{v}} + i\underline{\boldsymbol{\beta}}_{\boldsymbol{F}} = \underline{\boldsymbol{v}}/\boldsymbol{c} + i\underline{\boldsymbol{F}}/\boldsymbol{F_G}$,
$\underline{\boldsymbol{v}}$ – is a const velocity
$\underline{\boldsymbol{F}}$ – is a const force, $\boldsymbol{F_G} = \boldsymbol{c}^4/4\boldsymbol{G_N}$ – max force

$\boldsymbol{\Gamma} = \det \boldsymbol{\Lambda}^{-1}(\boldsymbol{\beta})$, $\quad \boldsymbol{I_3} = \mathrm{diag}\{\mathbf{1}, \mathbf{1}, \mathbf{1}\}$, $\quad \underline{\boldsymbol{\beta}} \cdot \underline{\boldsymbol{\beta}}^*$ – $\mathbf{3} \times \mathbf{3}$ tensor-diada.

$$\boldsymbol{Z}' = \boldsymbol{\Lambda}(\boldsymbol{\beta}) \boldsymbol{Z} : \quad \begin{pmatrix} \boldsymbol{Q}_0' + i\boldsymbol{P}_0' \\ \underline{\boldsymbol{Q}}' + i\underline{\boldsymbol{P}}' \end{pmatrix} = \boldsymbol{\Lambda}(\boldsymbol{\beta}) \begin{pmatrix} \boldsymbol{Q}_0 + i\boldsymbol{P}_0 \\ \underline{\boldsymbol{Q}} + i\underline{\boldsymbol{P}} \end{pmatrix}$$

The simplest case $\underline{\boldsymbol{\beta}}_{\boldsymbol{v}} = \{\boldsymbol{\beta_v}, \mathbf{0}, \mathbf{0}\}$, $\underline{\boldsymbol{\beta}}_{\boldsymbol{F}} = \{\boldsymbol{\beta_F}, \mathbf{0}, \mathbf{0}\}$

$$\boldsymbol{\Lambda(\beta) = \Gamma(\beta) \begin{pmatrix} 1 & \beta \\ \beta^* & 1 \end{pmatrix}, \quad \beta = \beta_v + i\beta_F,}$$

$$\boldsymbol{\Gamma(\beta) = \frac{1}{\sqrt{1 - |\underline{\beta}|^2}}, \quad \beta_v = \frac{v}{c}, \quad \beta_F = \frac{F}{F_G},}$$

$\boldsymbol{F}$ – is the constant force uniformly accelerated (noninertial) $\boldsymbol{FR}$.

$$\boldsymbol{\begin{pmatrix} Q_0' + iP_0' \\ Q' + iP' \end{pmatrix} = \Gamma(\beta) \left\{ \begin{pmatrix} 1 & \beta_v \\ \beta_v^* & 1 \end{pmatrix} \right.}$$

$$\boldsymbol{\left. + i \begin{pmatrix} 0 & \beta_F \\ -\beta_F & 0 \end{pmatrix} \right\} \begin{pmatrix} Q_0 + iP_0 \\ Q + iP \end{pmatrix}}$$

$$\boldsymbol{\Gamma(\beta) = \left(1 - \beta_v^2 - \beta_F^2\right)^{-1/2}}$$

$$\beta_F = 0$$

$$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \gamma_v \begin{pmatrix} 1 & \beta_v \\ \beta_v & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix},$$

$$\begin{pmatrix} E'/c \\ p' \end{pmatrix} = \gamma_v \begin{pmatrix} 1 & \beta_v \\ \beta_v & 1 \end{pmatrix} \begin{pmatrix} E/c \\ p \end{pmatrix},$$

$$\gamma_v = 1/\sqrt{1-\beta_v^2}$$

$$\dot{x}' = \frac{\dot{x}+V}{1+\dot{x}V/c^2}:$$

$$\beta_v' = \frac{\beta_v+\beta_V}{1+\beta_v\beta_V}$$

$$\beta_v = 0, \quad F_G = \frac{c^4}{4G_N}$$

$$\begin{pmatrix} t' \\ p'/F_G \end{pmatrix} = \gamma_F \begin{pmatrix} 1 & -\beta_F \\ -\beta_F & 1 \end{pmatrix} \begin{pmatrix} t \\ p/F_G \end{pmatrix},$$

$$\begin{pmatrix} E' \\ F_G x' \end{pmatrix} = \gamma_F \begin{pmatrix} 1 & -\beta_F \\ -\beta_F & 1 \end{pmatrix} \begin{pmatrix} E \\ F_G x \end{pmatrix},$$

$$\gamma_F = 1/\sqrt{1-\beta_F^2}$$

$$\dot{p}' = \frac{\dot{p}-F}{1-\dot{p}F/F_G^2}:$$

$$\beta_f' = \frac{\beta_f-\beta_F}{1-\beta_f\beta_F},$$

$$\dot{p} = \frac{dp}{dt} = f, \quad \frac{dE}{dx} = -f$$

Hamiltonian selfreciprocal one-particle classical model

The selfreciprocal expression of Born interval

$$\boldsymbol{S_B^2 = \left(x_0^2 - \underline{r}^2\right)/q_e^2 + \left(p_0^2 - \underline{p}^2\right) = 0}$$

is by definition the Energy equation in the $(\mathbf{2+2\cdot 3})$-dimensional extended Phase Space (QTPH) [J.Synge, Class. Dyn.] and

$$\boldsymbol{P_0^2 + T^2 - \left(\underline{P}^2 + \underline{Q}^2\right)}$$

$$\boldsymbol{= \left(P_0 + \sqrt{\underline{P}^2 + \underline{Q}^2}\right)\left(P_0 - \sqrt{\underline{P}^2 + \underline{Q}^2}\right) = 0}$$

is the energy surface equation. According to $\boldsymbol{(QTPH)}$ definition $\boldsymbol{\mathcal{H}(\underline{Q}, \underline{P}, T) \equiv -P_0}$ is the Hamilton function of system. In classical version we are to choose the positive sign.

So that Hamilton function $\mathcal{H}(Q_k, P_k, T)$ is as follows

$$\mathcal{H}(Q_k, P_k, T) = \sqrt{H_0^2 - T^2}, \quad Q_k = \frac{x_k}{q_e}, \quad P_k = \frac{p_k}{p_e},$$

where

$$T = x_0/q_e = v_e t \quad (v_e = v(m_e) = æ_0/m_e)$$

$$H_0 = \sqrt{P_k^2 + Q_k^2} \text{ – energy (integral of motion)}$$

$$\frac{dH_0}{dT} = \left\{H_0(\underline{P}, \underline{Q}), \mathcal{H}(\underline{P}, \underline{Q}, T)\right\}_{q,p} = 0$$

The Hamiltonian equations

$$\frac{dQ_k}{dT} = \frac{P_k}{\sqrt{H_0^2 - T^2}}, \quad \frac{dP_k}{dT} = \frac{-Q_k}{\sqrt{H_0^2 - T^2}}$$

possess under the spherical-symmetric initial conditions the following solution

$$|\vec{Q}| = \sin(\varphi + \varphi_{min}), \quad |\vec{P}| = \cos(\varphi + \varphi_{min}),$$

$$\varphi = \arcsin(T/H_0) \quad (0 < \varphi < \pi/2),$$

$$\varphi_{min} \neq 0 \text{ – minimal angle} \sim \text{minimal value of} \quad p_e q_e$$

The model: massive pulsating sphere

Frequency of pulsation (comoving $\boldsymbol{FR}$ time)

$$\omega_0(m_e) = \frac{æ_0}{H_0 m_e} = \frac{c^3}{4 m_e G_N} \sim \frac{10^{38}\mathrm{s}^{-1}}{m_e}.$$

Pulsation Period

$$T_0(m_e) = 2\pi/\omega_0(m_e) \sim m_e \cdot 10^{-38}\mathrm{s}.$$

Spatial peak value

$$R(m_e) = 4 m_e G_N / c^2 \sim m_e \cdot 10^{-28}\mathrm{sm}.$$

For $m_e = m_u \sim 10^{55}$ gr,

$$T(m_u) \sim 10^{17}\mathrm{s} \sim H_0^{-1} \sim \Delta t_u, \quad R(m_u) \sim 10^{27}\mathrm{sm} \sim R_u$$

.

Elliptic trajectory: $r - p$-spase

The eccentricity

$$\varepsilon(\alpha) = \frac{\sqrt{\vec{Q}_+^2 - \vec{Q}_-^2}}{\left|\vec{Q}_+\right|} = \frac{\sqrt{2}\left(1-\alpha^2\right)^{1/4}}{\left(1+\sqrt{1-\alpha^2}\right)^{1/2}}$$

$$\varepsilon(\alpha)|_{\alpha \ll 1} \cong 1 - \frac{\alpha^2}{8} = 1 - \frac{\vec{Q}^2 \vec{P}^2}{2\left(\vec{Q}^2 + \vec{P}^2\right)^2}, \quad \text{bat} \neq 1.$$

$$\alpha^2|_{P \ll Q} \cong \frac{4\vec{P}^2}{\vec{Q}^2} = \frac{2p}{r æ_0}, \quad \frac{p}{r} \ll æ_0$$

$P(\dot{Q})$ – *dependence*

$$\dot{\vec{Q}} = \frac{d\vec{Q}}{dT} = \frac{\vec{P}}{\sqrt{\vec{P}^2 + \vec{Q}^2 - T^2}} \to \vec{P}^2 = \frac{\dot{\vec{Q}}^2}{1 - \dot{\vec{Q}}^2}\left(\vec{Q}^2 - T^2\right)$$

$\vec{Q}^2 - T^2 = 1$ – hyperbolic motion!

$$\vec{P}^2_{max} = \frac{\dot{\vec{Q}}^2_{max}}{1 - \dot{\vec{Q}}^2_{max}}. \quad \text{But} \quad |\vec{P}_{max}| = 1$$

Therefore $2\dot{\vec{Q}}^2_{max} = 1$ and $\dot{r}_{max} = c/\sqrt{2}$

The case $S_B^2 = \Lambda_B \neq 0$

From $S_B^2 = P^\mu P_\mu + Q^\mu Q_\mu = \Lambda_B > 0$ to
$\bar{S}_B^2 = S_B^2 - \Lambda_B = 0$.
The Energy Integral is:

$$H_0(P,Q)|_{\Lambda_B=0} \to H_0(P,Q,\Lambda_B)$$

$$= H_0 = \sqrt{\vec{P}^2 + \vec{Q}^2 + \Lambda_B}$$

$$p_e(m), \quad q_e(m), \quad \frac{p_e(m)}{q_e(m)} = æ_0$$

$$\to p_e(M), \quad q_e(M), \quad \frac{p_e(M)}{q_e(M)} = æ_0$$

Scale transformation of the parameter: $M = m\sqrt{1+\Lambda_B}$.
Consider the case $\Lambda_B = 1$.
Two versions: Classical and Quantum

The Newtonian Limit

Let us consider the Energy integral $H_0 = \sqrt{\underline{P}^2 + \underline{Q}^2 + 1}$ under conditions $|\underline{Q}| \ll 1, \quad |\underline{P}| \ll 1, \quad |\underline{P}|/|\underline{Q}| \ll 1$.
Use the dimensional values

$$E = cp_e H_0 = cp_e \left(\frac{p^2}{p_e^2} + \frac{r^2}{q_e^2} + 1 \right)^{1/2}$$

$$= cp_e \sqrt{1 + \frac{r^2}{q_e^2}} \left\{ 1 + \frac{p^2}{p_e^2} \left(1 + \frac{r^2}{q_e^2} \right)^{-1} \right\}^{1/2}$$

$$\approx mc^2 + \frac{1}{2} mv^2 + \frac{1}{2} mr^2 \omega_0^2(m).$$

Omitting the “rest energy” mc^2 we receive

$$E_N = \frac{1}{2} mv^2 + \frac{1}{2} I_r \omega_0^2(m)$$

where $I_r = mr^2$ – moment of inertia
$\omega_0 = æ_0/m$ – angular velocity
$E_{rot} = mr^2\omega_0^2/2$– is the kinesthetic energy of rotation.

$Canonical\ Quantization\ S_B^2 \to \hat{S}_B^2$

Accoding the universal condition

$$m_e^2 = \frac{ca_e}{4\pi G_N} \to \left(a_e = \frac{h}{2}\right) \to \frac{\hbar c}{4G_N} = \frac{1}{4}M_P^2$$

the parameters p_e, q_e, E_e, t_e becomes Planken. The basic Born equation can be written in the form (Fock presentation)

$$\hat{S}_B|n_0, n\rangle = 2\left(\hat{H}_0 - \sum_{k=1}^{3}\hat{H}_k\right)|n_0, n\rangle = \lambda_B(n_0, n)|n_0, n\rangle,$$

where $\hat{H}_\mu$ $(\mu = 0, 1, 2, 3)$ are linear $QHOsc$ Hamiltonians.

The discrete spectrum of $(\text{mass})^2$:

$$M^2(n_0, n) = \{n_0 - (n+1)\}\, M_P^2 = N(n_0, n)\left(\frac{\hbar c}{G_N}\right)$$

where $N(n_0, n) = n_0 - (n+1)$, $n = \sum_{k=1}^{3} n_k$.
Each of $n_\mu = 0, 1, 2, \ldots$ $(\mu = 0, 1, 2, 3)$ independently.
Eigenstates

$$|n_0, n\rangle = \prod_{\mu=0}^{3} |n_\mu\rangle, \quad N(n) = \frac{1}{2}(n+1)(n+2)$$

is the degeneracy degree of $|n\rangle \sim |n_1\rangle|n_2\rangle|n_3\rangle$ states. In reality the operator $\hat{S}_B^2$ is looking out as the quantum-mechanical action operator having linear spectrum S_N in unit of h:

$$S_N = N\hbar \quad (N = 1, 2, 3, \ldots)$$

Canonical Quantization $H_0(\underline{P},\underline{Q}) \to \hat{H}_0(\underline{\hat{P}},\underline{\hat{Q}})$

Let as consider the variables $\hat{P}_k$, $\hat{Q}_l$ in the classical Energy integral

$$H_0 = \sqrt{\underline{P}^2 + \underline{Q}^2 + 1}$$

as canonically conjugated operators $[\hat{P}_k, \hat{Q}_l] = -i\delta_{kl}$, the constants p_e and q_e in definition

$$\hat{Q}_l = \frac{\hat{x}_l}{q_e}, \quad \hat{P}_k = \frac{\hat{p}_k}{p_e}$$

are Planken.

$$\hat{H}_0 \to \hat{H}_0 = \left(\underline{\hat{P}}^2 + \underline{\hat{Q}}^2 + 1\right)^{1/2}$$

The root extraction according Dirac procedure gives (in noncovariant notations):

for $\boldsymbol{DO}$ Hamiltonian and its Supersymmetry Partner $\boldsymbol{H^{\pm}_{DO}}$

$$\boldsymbol{\hat{H}^{\pm}_{0}(\hat{P}, \hat{Q}, 1) = \alpha_k \hat{P}_k + \beta \left(I \mp i\alpha_k \hat{Q}_k \right) =}$$

$$\boldsymbol{= \hat{H}^{\pm}_{DO} = \alpha_k \left(\hat{P}_k \mp i\beta \hat{Q}_k \right) + \beta,}$$

where $\boldsymbol{\alpha_k}$, $\boldsymbol{\rho_k}$ $\boldsymbol{(k = 1, 2, 3)}$ – $\boldsymbol{4 \times 4}$ Dirac matrices, $\boldsymbol{\beta = \rho_3}$.
$\boldsymbol{\hat{H}^{\pm}_{DO} = \hat{H}^{\pm}_{DO}(M_{Pl})}$ – well known Dirac Oscillator Hamiltonian which describes the one-half Spin particle with a Plank mass. The exact solutions of this model are well known. But generally accepted physical interpretation of $\boldsymbol{DO}$-model is absent up to now.

Thank you for your attention